DATE : 13/05/2012	**IDEAL TEST SERIES**	**MARKS : 30**
STD : X	**SUBJECT: ALGEBRA (SOL.)**	**TIME :1 HR**

Ans.1. Attempt any four from the following sub-questions: **[4 M]**

(1) $t_n = 3n + 1$

$n = 1, \quad t_1 = 3(1) + 1 = 3 + 1 = 4$

$n = 2, \quad t_2 = 3(2) + 1 = 6 + 1 = 7$

$n = 3, \quad t_3 = 3(3) + 1 = 9 + 1 = 10$

$n = 4, \quad t_4 = 3(4) + 1 = 12 + 1 = 13$

$n = 5, \quad t_5 = 3(5) + 1 = 15 + 1 = 16$

First Five terms of sequence are 4, 7, 10, 13, 16.

(2) Here

A = 25

B = 46

We, know that

$A = \frac{x+y}{2}$

$25 = \frac{x+46}{2}$

$50 = x + 46$

$\boxed{X = 4}$

(3) $1^3, 2^3, 3^3, 4^3, ……..$

Here $t_1 = 1, t_2 = 8, t_3 = 27, t_4 = 64$

$d = t_2 - t_1 = 8 - 1 = 7$

$d = t_3 - t_2 = 27 - 8 = 19$

Here difference between two consecutive terms is not constant. Hence the given sequule is not an A r Y.y

(4) Here $t_1 = a = 9, d = 2$

$t_2 = t_1 + d = 9 + 2 = 11$

$t_3 = t_2 + d = 11 + 2 = 13$

$t_4 = t_3 + d =- 13 + 2 =15$

The First four consecutive term of A. P are 9, 11, 13, 15

(5) Here a = 3 , r = 2

$t_n = a.r^{n-1}$

n = 2

$t_2 = 3 \times 2^{2-1}$

$= 3 \times 2$

$= 6$

$t_2 = 6$

Ans.2. Attempt any five sub-questions from the following sub-question

[10 M]

(1) Here a = 6, d = 3

$S_n = \frac{n}{2}$ [2a + 1 n – 11 d] Formula

Pat n = 10

$S_{10} = \frac{10}{2} [2 \times 6 + (10 - 1) 3]$

$= 5 [12 + 9 \times 3]$

$= 5 [12 + 27]$

$= 5 \times 39$

$S_{10} = 195$

(2) Sequence 122, 116, 110…………….

Here a = 122, d = 116 – 122 = -6

$t_n = a + (n - 1) d$

$= 122 + (n - 1) - 6$

$= 122 - 6n + 6$

$= 128 - 6n$

But tn < 0 (given)

$128 - 6n < 0$

$128 < 6n$

$\frac{128}{6} < n$

$21 \frac{2}{6} < n$

$\therefore$ n = 22 (smallest n)

(3) Let

G be Geometric mean of

$\sqrt{82}$ - 1 and $\sqrt{82}$ + 1

We know that

$G = \pm \sqrt{xy}$

$G = \pm \sqrt{(\sqrt{82} - 1)(\sqrt{82} + 1)}$

$G = \pm \sqrt{(\sqrt{82})^2 - (1)^2}$

$G = \pm \sqrt{82 - 1}$

$G = \pm \sqrt{81}$

$G = \pm 9$

Geometric mean is 9

(4) 12, 16, 20, 24, ………..

Here

a= 12, d = 16 – 12 = 4

$t_n = a + (n - 1) d$ (formula)

n = 25

$t_{25} = 12 + (25 - 1) 4$

$= 12 + 24 \times 4$

$= 12 + 96$

$= 108$

$\therefore$ twenty fifth term of A.P is 108

(5) Let the three terms be $\frac{a}{r}$, a, ar

By First condition

$\frac{a}{r} + a = 9$ (1)

By Second condition

$\frac{a}{r} \times a \times ar = 216$

$a^3 = 216$

a = 6

pat a = 6 1 n equ (1)

$\frac{6}{r} + 6 = 9$

$\frac{6}{r} = 9 - 6$

$\frac{6}{r} = 3$

$\boxed{r = 2}$

Thus three terms $\frac{a}{r}$, a, ar

are , $\frac{6}{r}$, 6, 6×2

i.e 3, 6, 12

$\therefore$ The three consecutive terms are 3, 6, 12

(6) 1, 3, 9, ……….

Here a = 1, $r = \frac{t_2}{t_1} = \frac{3}{1} = 3$

$S_n = \frac{a(r^n - 1)}{(r-1)}$ (r > 1)

n = 7

$S_7 = \frac{1(3^7 - 1)}{(3-1)}$

$= \frac{2187-1}{2} \quad \frac{2187}{2} = 1093$

$\boxed{S_7 = 1093}$

Ans.3. Attempt any four from following sub-question. [12 M]

(1) $S_3 = 31, \quad S_6 = 3906$

Let a be the first term and r be the common Ration

$S_n = \frac{a(r^n-1)}{(r-1)}$

(i) n = 3

$S_3 = \frac{a(r^3-1)}{(r-1)}$

$31 = \frac{a(r^3-1)}{(r-1)}$ (1)

(ii) n = 6

$S_6 = \frac{a(r^6-1)}{(r-1)}$

$3906 = \frac{a(r^6-1)}{(r-1)}$ (2)

Dividing equ (2) by (1)

$\frac{3906}{31} = \frac{a(r^6-1)}{(r-1)} \div \frac{a(r^3-1)}{(r-1)}$

$126 = \frac{a(r^6-1)}{(r-1)} \times \frac{(r-1)}{a(r^3-1)}$

$126 = \frac{(r^3)^2-(1)^2}{r^3-1}$

$126 = \frac{\cancel{(r^3-1)}(r^3+1)}{\cancel{(r^3+-1)}}$

$126 - 1 = r^3 + 1$

$125 = r^3$

r = 5 (cube roots)

Put r = 5 in equ (1)

$31 = \frac{a(5^3-1)}{5-1}$

$31 = \frac{a\,(125-1)}{4}$

$31 = \frac{a \times \cancel{124}^{31}}{\cancel{4}}$

$\boxed{a = 1}$

(2) The two digit number when divided by number 5 learc reminder 1 are

11, 16, 21,..........................96

$a = 11, d = 5, t_n = 96$

$t_n = a + (n - 1) d$

$96 = 11 + (n - 1) 5$

$96 - 11 = 5n - 5$

$85 + 5 = 5n$

$85 = 5n$

$\boxed{n = 18}$

There are 18 two – digit numbers which leaves remainder 1 when divided by 5.

(3) Let a be the first term and d be the common difference Number of instalment (n) = 10 each instalment being less than the preceding one by - 10

$d = -10$

S_{10} = Total amount to be repaid

= 4000 + 500

= 4500

$S_n = \frac{n}{2} [2a + 1 (n - 1) d]$

$S_{10} = \frac{10}{2} [2 \times a + (10 - 1) \times -10]$

$4500 = 5 [2a - 90]$

$\frac{4500}{5} 2a - 90$

$900 + 90 = 2a$

$2a = 990$

$a = 495$

First instalment = 495

<u>Last instalment (t_n)</u>

$t_n = a + (n - 1) d$

$t_{10} = 495 + (10 - 1) - 10$

$= 495 + 9 \times - 10$

$= 495 - 90$

$= 405$

First and last instaments are $t_1 =$ 495 and $t_n =$ 405 respectively.

(4) (i) 1, 2, 3, 4,..................n

Here a = 1, d = 2 – 1 whihch are in A.P

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$= \frac{n}{2}$ [2 × 1 + (n – 1) 1]

$= \frac{n}{2}$ [2 + n – 1]

$= \frac{n}{2}$ [n + 1] (1)

(ii) x, x^2, x^3 ………………..x^{n-1}

Here

a = x, r = x which are in G. P

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{x(x^n - 1)}{(x-1)}$ (ii)

Sum of all terms (S_n) [Adding (1) and (ii)]

$= \frac{n}{2}(n + 1) + \frac{x(x^n - 1)}{x-1}$

(5) $S_{55} = 3300$

We know that

$S_n = \frac{n}{2}$ [2a + (n – 1) d]

$S_{55} = \frac{55}{2}$ [2a + (55 – 1) d]

$3300 = \frac{55}{2}$ [2a + 54 d]

$3300 = \frac{55}{\not{2}} \times \not{2}$ (a + 27d}

$\frac{3300}{55}$ = a + 27 d

a + 27 d = 60 (1)

Now , for n = 28

tn = a + (n – 1) d

t_{28} = a + (28 – 1) d

= a + 27 d

= 60 [from equ (1)]

The 28th term is 60.

Ans.4. Attempt any one from the following sub-question. [4 M]

(1) Runs scored by sachin, sehwag and Dhoni are in G.P.

Let $\frac{a}{r}$, a, ar be runs scored by Sachin, Sehwag and Dhoni respectively

By 1st Condition

$\frac{a}{r} + a + ar = 228$ (1)

By 2nd Condition

$a + ar = \frac{a}{r} + 12$

$-\frac{a}{r} + a + ar = 12$ (2)

Sabtracting equ (2) from (1)

$\frac{a}{r} + a + ar = 228$

$-\frac{a}{r} + a + ar = 12$

$+ \quad - \quad - \quad -$

$\frac{2a}{r} = 216$

$\frac{a}{r} = 108$

$\boxed{a = 108r}$ (3)

Put a = 108r in equ (1), we get

$108 + 108r + 108\, r^2 = 228$

$108\, r^2 + 108r + 108 - 228 = 0$

$108\, r^2 + 108r - 120 = 0$

$9r^2 + 9r - 10 = 0$ (Dividing equ by 108)

$9r^2 + 15r - 6r - 10 = 0$

$3r\,(3r + 5) - 2\,(3r + 5) = 0$

$(3r + 5)\,(3r - 2) = 0$

$3r + 5 = 0$ or $3r - 2 = 0$

$r = \frac{-5}{3}$ $\quad$ $r = \frac{2}{3}$

Ratio of runs scored can not be Negative

$r = \frac{2}{3}$

Putting $r = \frac{2}{3}$ in equation (3)

$a = 108 \times \frac{2}{3} = 72$

$\therefore$ Sachin scored $= \frac{a}{r} = 108$

Sehwag scored = a = 72

Dhoni scored = ar= $72 \times \frac{2}{3}$

$= 48$

Sachin, Sehwag, and Dhoni scored 108, 72, 48 runs respectively.

(2) 7, 14, 21,................................

Here a = 7, d = 14-7 = 7

$S_n = 5740$

$S_n = \frac{n}{2} [2a + (n - 1) d]$

$5740 = \frac{n}{2} [2 \times 7 + (n - 1) 7]$

$11480 = n [14 + 7n - 7]$

$11480 = 14 n + 7n^2 - 7n$

$11480 = 7n^2 + 7n$

$7n^2 + 7n - 11480 = 0$ (Dividing equ by 7)

$n^2 + n - 1640 = 0$

$n^2 + 41n - 40n - 1640 = 0$

$n(n + 41) - 40 (n + 41) = 0$

$(n + 41) (n - 40) = 0$

$n + 41 = 0$ or $n - 40 = 0$

$n = -41$ or $n = 40$

Numbers of terms can not be –ve

$n = 40$

Sum of 40 terms is 5740.